B K BIRLA CENTRE FOR EDUCATION

SARALA BIRLA GROUP OF SCHOOLS
SENIOR SECONDARY CO-ED DAY CUM BOYS' RESIDENTIAL SCHOOL
PRE BOARD - II (2024-25)
MATHEMATICS (041)
MARKING SCHEME

Section A

1.

(d) 512

Explanation:

Since each element a_{ij} can be filled in two ways (with either '2' or "0'), total number of possible matrices is $8 \times 8 \times 8 = 512$

2.

(d) Null matrix

Explanation:

As we know that, A(adj A) = |A| I.

But it is given that A is a singular matrix

Thus, |A| = 0.

Therefore, A(adj A) = 0I = 0, where 0 is the zero matrix.

Hence, if A is a singular matrix, then A(adj A) = $\underline{\mathbf{0}}$.

3.

(d) q = 0, s = - 4

Explanation:

We have

$$\Delta = \begin{vmatrix} x^2+x & 2x-1 & x+3 \\ 3x+1 & 2+x^2 & x^3-3 \\ x-3 & x^2+4 & 2x \end{vmatrix} = px^7 + qx^6 + rx^5 + sx^4 + fx^3 + ux^2 + vx + w$$

$\Rightarrow (x^2 + x) \{(4x + 2x^3) - (x^5 + 4x^3 - 3x^2 - 12)\} - (3x + 1) \{(4x^2 - 2x) - (x^3 + 3x^2 + 4x + 12)\} + (x - 3) \{(2x^4 - x^3 - 6x + 3) - (x^3 + 3x^2 + 2x + 6)\}$

$= px^7 + qx^6 + rx^5 + sx^4 + fx^3 + ux^2 + vx + w$

$\Rightarrow -x^7 - x^6 + 0x^5 - 4x^4 + 8x^3 + 34x^2 + 75x + 21 = px^7 + qx^6 + rx^5 + sx^4 + fx^3 + ux^2 + vx + w$

p = - 1, q = - 1, r = 0, s = - 4, t - 8, u - 34, v = 75, w = 21

4.

(d) $-2 \tan\left(\frac{3x+4}{5x+6}\right) \times \frac{1}{(5x+6)^2}$

Explanation:

$-2 \tan\left(\frac{3x+4}{5x+6}\right) \times \frac{1}{(5x+6)^2}$

5.

(c) 90°

Explanation:

To find the angle with the z-axis, we use the relation:

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

where $\alpha = 30°$ and $\beta = 120°$. Calculating, we get:

$\cos^2 30° = \frac{3}{4}, \cos^2 120° = \frac{1}{4}$

Thus,

$\frac{3}{4} + \frac{1}{4} + \cos^2\gamma = 1 \Longrightarrow \cos^2\gamma = 0 \Longrightarrow \gamma = 90°.$

So, the angle with the z-axis is 90°.

6. (c) 3

Explanation:

3

7.

(c) linear function

Explanation:

linear function

8.

(b) -2

Explanation:

Given $\vec{a} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \lambda\hat{k}$ such that $\vec{a} \perp \vec{b}$

$\therefore 6 - 8 - \lambda = 0 = -2 - \lambda = 0$

$\Rightarrow \lambda = -2$

9.

(d) tan x

Explanation:

tan x

10.

(c) 4

Explanation:

Here, A = $\begin{bmatrix} 5 & 8 & 11 \\ 4 & 5 & 13 \\ 7 & 5 & 5 \end{bmatrix}$

Thus, number of elements more than 5, is 4.

11.

(d) Option (c)

Explanation:

If a LPP admits two optimal solutions it has an infinite solution.

12.

D)

Cosθ = 3/5

$\therefore sin\theta = \frac{4}{5}$

$|\vec{a}x\vec{b}|$ = $|a||b|$sinθ =16

13.

(c) PX = -X

Explanation:

Given P' = 2P + I

$\Rightarrow$ (P')' = (2P + I)' = 2P' + I' = 2P' + I

$\Rightarrow$ P = 2(2P + I) + I = 4P + 2I + I

$\Rightarrow$ P = 4P + 3I $\Rightarrow$ -3P = 3I $\Rightarrow$ P = -I

$\therefore$ PX = -IX = -X

14. **(a)** $\frac{3}{28}$

Explanation:

Required probability $= \frac{3}{8} \cdot \frac{2}{7} = \frac{3}{28}$

15.

(d) $e^y - e^x = \frac{x^3}{3} + C$

Explanation:

We have, $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$

$\Rightarrow e^y\, dy = (e^x + x^2)dx$

$\Rightarrow \int e^y dy = \int \left(e^x + x^2\right) dx$

$\Rightarrow e^y = e^x + \frac{x^3}{3} + c$

$\Rightarrow e^y - e^x = \frac{x^3}{3} + c$

16.

(d) $\frac{1}{\sqrt{6}}$

Explanation:

$\frac{1}{\sqrt{6}}$

17.

(b) 1.5

Explanation:

[x] is always continuous at non-integer value of x. Hence, f(x) = [x] will be continuous at x = 1.5.

18.

(b) 10

Explanation:

Determinant of these point should be zero

$$\begin{vmatrix} -1 & 3 & 2 \\ -4 & 2 & -2 \\ 5 & 5 & \lambda \end{vmatrix} = 0$$

$-1(2\lambda + 10) - 3(-4\lambda + 10) + 2(-20 - 10) = 0$

$10\lambda = 10 + 30 + 60 = 100$

$\lambda = 10$

19.

(d) A is false but R is true.

Explanation:

Let $f(x) = 2x^3 - 24x$

$\Rightarrow f'(x) = 6x^2 - 24 = 6(x^2 - 4)$

$= 6(x + 2)(x - 2)$

For maxima or minima put $f'(x) = 0$.

$\Rightarrow 6(x + 2)(x - 2) = 0$

$\Rightarrow x = 2, -2$

We first consider the interval [1, 3].

So, we have to evaluate the value of f at the critical point $x = 2 \in [1, 3]$ and at the end points of [1, 3].

At x = 1, $f(1) = 2 \times 1^3 - 24 \times 1 = -22$

At x = 2, $f(2) = 2 \times 2^3 - 24 \times 2 = -32$

At x = 3, $f(3) = 2 \times 3^3 - 24 \times 3 = -18$

$\therefore$ The absolute maximum value of f(x) in the interval [1, 3] is -18 occurring at x = 3.

Hence, Assertion is false and Reason is true.

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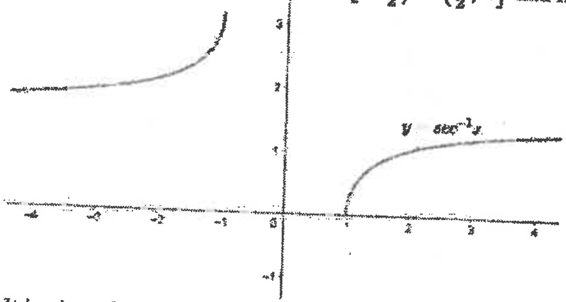
20. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:
For one to one function, if f(x) = f(y)
then x = y
$\therefore 1 + x_1^2 = 1 + x_2^2$
$x_1^2 = x_2^2$
here, every element in the range maps to only one element in domain.
∴ f(x) is strictly monoatomic function and one to one function.

Section B

21. We know that the range of principal value of cosec^{-1} is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ - [0]
Let $\text{cosec}^{-1}(-1) = \theta$. Then we have, cosec θ = -1
cosec θ = -1 = -cosec $\frac{\pi}{2}$ = cosec $\left(\frac{-\pi}{2}\right)$
$\therefore \theta = \frac{-\pi}{2} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ - [0]
Hence, the principal value of $\text{cosec}^{-1}(-1)$ is equal to $\frac{-\pi}{2}$

OR

Principal value branch of $\sec^{-1} x$ is $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$ and its graph is shown below.



22. It is given that f(x) = |x + 2| - 1
Now, we can see that |x + 2| ≥ 0 for every x ∈ R
$\Rightarrow f(x) = |x + 2| - 1 \geq -1$ for every x ∈ R
Clearly, the minimum value of f is attained when |x + 2| = 0
i.e, |x + 2| = 0
⇒ x = -2
Then, Minimum value of f = f(-2) = |-2 + 2| - 1 = -1
Therefore, function f does not have a maximum value.

23. Here
f(x) = sin x + sin x cos x
$\Rightarrow f'(x) = \cos x + \sin x(-\sin x) + \cos x \cos x$
$\Rightarrow f'(x) = \cos x - \sin^2 x + \cos^2 x$
$\Rightarrow f'(x) = \cos x + \cos^2 x - 1 + \cos^2 x$
$\Rightarrow f'(x) = 2\cos^2 x + \cos x - 1$
$\Rightarrow f'(x) = 2\cos^2 x + 2\cos x - \cos x - 1$
⇒f'(x) = 2 cos x(cos x + 1) - 1(cos x + 1)
$\Rightarrow f'(x) = (2\cos x - 1\cos x + 1)$
for f(x) to be increasing, we must have
$f'(x) > 0$
$\Rightarrow (2\cos x - 1)(\cos x + 1) > 0$
This is only possible when
$(2\cos x - 1) > 0$ and $(\cos x + 1) > 0$
$\Rightarrow \cos x > \frac{1}{2}$ and $\cos x > -1$
$\Rightarrow x \in \left(0, \frac{\pi}{3}\right)$ and $x \in \left(0, \frac{\pi}{2}\right)$
$So, x \in \left(0, \frac{\pi}{3}\right)$

f(x) is increasing on $(0, \frac{\pi}{3})$

For f(x) to be decreasing we, must have

$\Rightarrow (2\cos x - 1)(\cos x + 1) < 0$

This is only possible when

(2cos x - 1) < 0 and (cos x + 1) > 0

⇒ (2cos x - 1) < 0 and (cos x + 1) > 0

$\Rightarrow \cos x < \frac{1}{2}$ and $\cos x > -1$

$\Rightarrow x \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$ and $x \in \left(0, \frac{\pi}{2}\right)$

$\therefore f(x)$ is decreasing on $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

OR

Let one of the numbers be x. Then the other number is (15 – x).

Let S(x) denote the sum of the squares of these numbers. Then

$S(x) = x^2 + (15 - x)^2 = 2x^2 - 30x + 225$

or $\begin{cases} S'(x) = 4x - 30 \\ S''(x) = 4 \end{cases}$

Now S'(x) = 0, gives, $x = \frac{15}{2}$.

Also $S''\left(\frac{15}{2}\right) = 4 > 0$.

Therefore, by second derivative test, $x = \frac{15}{2}$ is the point of local minima of S. Hence the sum of squares of numbers is minimum when the numbers are $\frac{15}{2}$ and $15 - \frac{15}{2} = \frac{15}{2}$.

24. Let $I = \int_0^{\pi} \frac{x}{\left(a^2\cos^2 x + b^2\sin^2 x\right)} dx$ .... (i)

Then, $I = \int_0^{\pi} \frac{(\pi - x)}{\left[a^2\cos^2(\pi - x) + b^2\sin^2(\pi - x)\right]} dx$

or $I = \int_0^{\pi} \frac{(\pi - x)}{\left(a^2\cos^2 x + b^2\sin^2 x\right)} dx$ .... (ii)

Adding (i) and (ii), we get

$2I = \int_0^{\pi} \frac{(x + \pi - x)}{\left(a^2\cos^2 x + b^2\sin^2 x\right)} dx = \pi \int_0^{\pi} \frac{dx}{\left(a^2\cos^2 x + b^2\sin^2 x\right)}$ [dividing num. and denim. by $\cos^2 x$]

$= 2\pi \int_0^{\infty} \frac{dt}{\left(a^2 + b^2t^2\right)}$, where tan x = t

$= \frac{2\pi}{b^2} \int_0^{\infty} \frac{dt}{\left(\frac{a^2}{b^2} + t^2\right)} = \left[\frac{2\pi}{b^2} \cdot \frac{b}{a}\tan^{-1}\left(\frac{bt}{a}\right)\right]_0^{\infty}$

$= \frac{2\pi}{ab}\left[\tan^{-1}(\infty) - \tan^{-1}(0)\right] = \frac{2\pi}{ab}\left(\frac{\pi}{2} - 0\right) = \left(\frac{2\pi}{ab} \times \frac{\pi}{2}\right) = \frac{\pi^2}{ab}$

$\therefore \quad I = \frac{\pi^2}{2ab} \Rightarrow \int_0^{\pi} \frac{x}{\left(a^2\cos^2 x + b^2\sin^2 x\right)} dx = \frac{\pi^2}{2ab}$

25. Given, $f(x) = \log_a x$

domain of f(x) is x > 0

$f'(x) = \frac{1}{x}\ln(a)$

⇒ for a > 1, ln(a) > 0,

hence f'(x) > 0 which means strictly increasing.

⇒ for 0 < a < 1, ln(a) < 0,

Therefore, f'(x) < 0 which means strictly decreasing.

Section C

26. We have, $\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = 1 - \frac{(4x^2+10)}{(x^2+3)(x^2+4)}$

Let $\frac{(4x^2+10)}{(x^2+3)(x^2+4)} = \frac{Ax+B}{(x^2+3)} + \frac{Cx+D}{(x^2+4)}$

$4x^2 + 10 = (Ax + B)(x^2 + 4) + (Cx + D)(x^2 + 3)$

$\Rightarrow 4x^2 + 10 = Ax^3 + 4Ax + Bx^2 + 4B + Cx^3 + 3Cx + Dx^2 + 3D$

$\Rightarrow 4x^2 + 10 = (A+C)x^3 + (B + D)x^2 + (4A + 3C)x + (4B + 3D)$

Equating the coefficients of x^3, x^2, x and constant term, we get,

A + C = 0

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B + D = 4

4A + 3C = 0

4B + 3 D = 10

On solving these equations, we get,

A = 0, B = -2, C = 0 and D = 6

Therefore,

$\frac{(4x^2+10)}{(x^2+3)(x^2+4)} = \frac{-2}{(x^2+3)} + \frac{6}{(x^2+4)}$

$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = 1 - \left(\frac{-2}{(x^2+3)} + \frac{6}{(x^2+4)}\right)$

$\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}dx = \int \left\{1 + \frac{2}{(x^2+3)} - \frac{6}{(x^2+4)}\right\} dx$

$= \left\{1 + \frac{2}{x^2+(\sqrt{3})^2} - \frac{6}{(x^2+2^2)}\right\}$

$= x + 2\left(\frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}\right) - 6\left(\frac{1}{2}\tan^{-1}\frac{x}{2}\right) + C$

$= x + \frac{2}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} - 3\tan^{-1}\frac{x}{2} + C$

27. A white ball can be drawn in two mutually exclusive ways:

i. By transferring a black ball from bag A to bag B, then drawing a white ball

ii. By transferring a white ball from bag A to bag B, then drawing a white ball

Consider the following events:

E_1 = A black ball is transferred from bag A to bag B

E_2 = A white ball is transferred from bag A to bag B

A = A white ball is drawn

Therefore,we have,

$P(E_1) = \frac{7}{15}$

$P(E_2) = \frac{8}{15}$

Now,

$P\left(\frac{A}{E_1}\right) = \frac{5}{10} = \frac{1}{2}$

$P\left(\frac{A}{E_2}\right) = \frac{6}{10} = \frac{3}{5}$

Using the law of total probability, we get

Required probability = $P(A) = P(E_1)P(\frac{A}{E_1}) + P(E_2)(\frac{A}{E_2})$

$= \frac{7}{15} \times \frac{1}{2} + \frac{8}{15} \times \frac{3}{5}$

$= \frac{7}{30} + \frac{8}{25}$

$= \frac{35+48}{150} = \frac{83}{150}$

28. Let the given integral be,

$\int_0^{\frac{\pi}{2}} |\sin x - \cos x| dx$

$= \sqrt{2}\int_0^{\frac{\pi}{2}} \left|\sin x\frac{1}{\sqrt{2}} - \cos x\frac{1}{\sqrt{2}}\right| dx$

$= \sqrt{2}\int_0^{\frac{\pi}{2}} |\sin x \cos\frac{\pi}{4} - \cos x \sin\frac{\pi}{4}| dx$

$= \sqrt{2}\int_0^{\frac{\pi}{2}} \left|\sin\left(x - \frac{\pi}{4}\right)\right| dx$

We have,

$$\left|\sin\left(x - \frac{\pi}{4}\right)\right| = \begin{cases} -\sin\left(x - \frac{\pi}{4}\right), & 0 \le x \le \frac{\pi}{4} \\ \sin\left(x - \frac{\pi}{4}\right), & \frac{\pi}{4} \le x \le \frac{\pi}{2} \end{cases}$$

$\therefore \int_0^{\frac{\pi}{2}} |\sin x - \cos x| dx = \sqrt{2}\int_0^{\frac{\pi}{4}} -\sin\left(x - \frac{\pi}{4}\right)dx + \sqrt{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin\left(x - \frac{\pi}{4}\right)dx$

$= \sqrt{2}\left[\cos\left(x - \frac{\pi}{4}\right)\right]_0^{\frac{\pi}{4}} - \sqrt{2}\left[\cos\left(x - \frac{\pi}{4}\right)\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= \sqrt{2}\left[\cos(0) - \cos\left(-\frac{\pi}{4}\right)\right] - \sqrt{2}\left[\cos\left(\frac{\pi}{4}\right) - \cos(0)\right]$

$= \sqrt{2}\left(1 - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} + 1\right)$

$= \sqrt{2}\left(2 - \frac{2}{\sqrt{2}}\right)$
$= 2\sqrt{2} - 2$
$= 2(\sqrt{2} - 1)$

OR

We can write Sin 2x = 2sinx.cosx

$\int e^{\sin x} \sin 2x\, dx = 2 \int e^{\sin x} . \sin x \cos x dx$

Let Sin x = t

Cos xdx = dt

$2 \int e^{\sin x} \sin x \cos x dx = 2 \int e^t\, t\, dt$

Using BY PARTS METHOD.

$2\int e^t \cdot t dt = 2\left[t\int e^t dt - \int\left(\frac{dt}{dt}\cdot\int e^t dt\right)dt\right]$
$= 2\left[t\cdot e^t - \int 1. e^t dt\right]$
$= 2\left[t\cdot e^t - e^t\right] + c$
$= 2e^t(t-1) + c$

Replacing t with sin x

$= 2e^{\sin x}(\sin x - 1) + c$

29. The given differential equation is,

$\frac{dy}{dx} = \frac{y(2y-x)}{x(2y+x)}$

$\Rightarrow \frac{dy}{dx} = \frac{y\left(2\frac{y}{x}-1\right)}{x\left(2\frac{y}{x}+1\right)}$

$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is:

Put y = vx

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{vx\left(2\frac{vx}{x}-1\right)}{x\left(2\frac{vx}{x}+1\right)} = v\left(\frac{2v-1}{2v+1}\right)$

$\Rightarrow x\frac{dv}{dx} = v\left(\frac{2v-1}{2v+1}\right) - v$

$\Rightarrow x\frac{dv}{dx} = \left(\frac{2v^2-v-2v^2-v}{2v+1}\right)$

$\Rightarrow x\frac{dv}{dx} = \frac{-2v}{2v+1}$

$\Rightarrow \frac{2v+1}{2v}dv = \frac{-dx}{x}$

$\Rightarrow dv + \left(\frac{1}{2v}\right)dv = \frac{-dx}{x}$

Integrating both the sides we get:

$\Rightarrow \int\left(dv + \left(\frac{1}{2v}\right)dv\right) = -\int\frac{dx}{x} + c$

$\Rightarrow v + \frac{\ln|v|}{2} = -\ln|x| + c$

Resubstituting the value of y = vx, we get,

$\Rightarrow \frac{y}{x} + \frac{\ln\left|\frac{y}{x}\right|}{2} = -\ln|x| + c$

y = 1 when x = 1

1 + 0 = -0 + c

⇒ c = 1

$\Rightarrow \frac{y}{x} + \frac{1}{2}\log|xy| = 1$

OR

The given differential equation is,

$y - x\frac{dy}{dx} = 2\left(1 + x^2\frac{dy}{dx}\right)$

$\Rightarrow y - 2 = 2x^2\frac{dy}{dx} + x\frac{dy}{dx}$

$\Rightarrow y - 2 = x\cdot(2x+1)\frac{dy}{dx}$

$\Rightarrow (y-2)\,dx = x\,(2x+1)\,dy$

$\Rightarrow \frac{1}{x(2x+1)}dx = \frac{1}{y-2}dy$

$\Rightarrow \int \frac{1}{x(2x+1)}dx = \int \frac{1}{y-2}dy$

$\Rightarrow \int \left(\frac{1}{x} - \frac{2}{2x+1}\right) dx = \int \frac{1}{y-2}dy$

$\Rightarrow \log |x| - \log |2x + 1| = \log |y - 2| + \log C$

$\Rightarrow \log\left|\frac{x}{2x+1}\right| = \log |y - 2| + \log C$

$\Rightarrow \log\left|\frac{x}{2x+1}\right| - \log |y - 2| = \log C$

$\Rightarrow \log\left|\frac{x}{2x+1} \times \frac{1}{y-2}\right| = \log C$

$\Rightarrow \left|\frac{x}{(2x+1)(y-2)}\right| = C$ ...(i)

It is given that y(1) = 1 i.e. y = 1 when x = 1. Putting x = 1 and y = 1 in (i), we get

$\left|-\frac{1}{3}\right| = C \Rightarrow C = \frac{1}{3}$

Putting $C = \frac{1}{3}$ in (i), we get

$\left|\frac{x}{(2x+1)(y-2)}\right| = \frac{1}{3}$

$\Rightarrow \frac{x}{(2x+1)(y-2)} = \pm\frac{1}{3}$

$\Rightarrow y - 2 = \pm\frac{3x}{2x+1} \Rightarrow y = 2 \pm \frac{3x}{2x+1}$

But, $y = 2 + \frac{3x}{2x+1}$ is not satisfied by y(1) = 1

Hence, $y = 2 - \frac{3x}{2x+1}$, where $x \neq -\frac{1}{2}$ is the required solution.

30. The given LPP can be re-written as:

Maximize or Minimize Z = 3x + 5y

Subject to

$3x - 4y \geq -12$

$2x - y \geq -2$

$2x + 3y \geq 12$

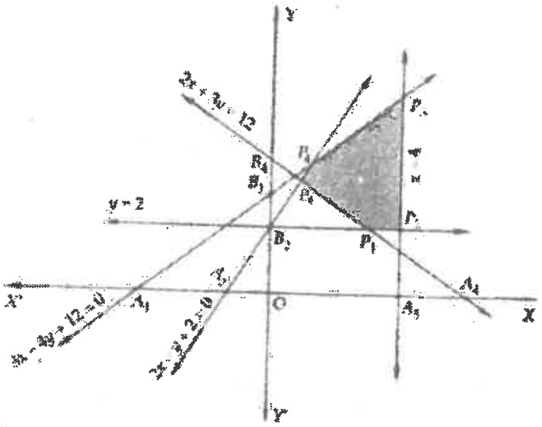
$x \leq 4$

$y \geq 2$

$x \geq 0$

Converting the inequations into equations, we obtain the following equations 3x - 4y = -1 2, 2x - y = - 2, 2x + 3y = 12, x = 4, y = 2 and x = 0.

These lines are drawn on suitable scale. The shaded region $P_1P_2P_3P_4P_5$ shown in Figure represents the feasible region of the given LPP.



The values of the objective function at these points are given in the following table:

Point (x, y)	Values of the objective function Z = 3x + 5y
$P_1(3, 2)$	$Z = 3 \times 3 + 5 \times 2 = 19$
$P_2(4, 2)$	$Z = 3 \times 4 + 2 \times 5 = 22$
$P_3(4, 6)$	$Z = 3 \times 4 + 5 \times 6 = 42$
$P_4(\frac{4}{5}, \frac{18}{5})$	$Z = 3 \times \frac{4}{5} + 5 \times \frac{18}{5} = \frac{102}{5}$

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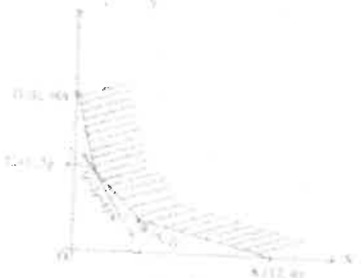
$P_5(\frac{3}{4}, \frac{7}{2})$	$Z = 3 \times \frac{3}{4} + 5 \times \frac{7}{2} = \frac{79}{4}$

Clearly Z assumes its minimum value 19 at x = 3 and y = 2. The maximum value of Z is 42 at x = 4 and y = 6.

OR

The feasible region (R) is unbounded. Therefore, a minimum of Z may or may not exist. If it exists, it will be at the corner point Fig.

Corner Point	Value of Z
A (12, 0)	3(12) + 2(0) = 36
B(4, 2)	3(4) + 2(2) = 16
C(1, 5)	3(1) + 2(5) = 13 (smallest)
D(0, 10)	3(0) + 2(10) = 20



Let us graph 3x + 2y < 13. We see that the open half plane determined by 3x + 2y < 13 and R do not have a common point. So, the smallest value 13 is the minimum value of Z.

31. Given function is

$$f(x) = \begin{cases} |x| + 3, & x \leq -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \geq 3 \end{cases} = \begin{cases} -x + 3, x \leq -3 \\ -2x, -3 < x < 3 \\ 6x + 2, x \geq 3 \end{cases}$$

First, we verify continuity at x = -3 and then at x = 3

Continuity at x = -3

LHL = $\lim_{x \to (-3)^-} f(x) = \lim_{x \to (-3)^-} (-x + 3)$

$\Rightarrow$ $LHL = \lim_{h \to 0} [-(-3 - h) + 3]$

$= \lim_{h \to 0} (3 + h + 3)$

= 3 + 3 = 6

and RHL = $\lim_{x \to (-3)^+} f(x) = \lim_{x \to (-3)^+} (-2x)$

$\Rightarrow$ $RHL = \lim_{h \to 0} [-2(-3 + h)]$

$= \lim_{h \to 0} (6 - 2h)$

$\Rightarrow$ RHL = 6

Also, f(-3) = value of f(x) at x = - 3

=-(-3) + 3

= 3 + 3 = 6

$\because$ LHL = RHL = f(-3)

$\therefore$ f(x) is continuous at x = - 3 So, x = - 3 is the point of continuity.

Continuity at x = 3

LHL = $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} [-(2x)]$

$\Rightarrow$ $LHL = \lim_{h \to 0} [-2(3 - h)]$

$= \lim_{h \to 0} (-6 + 2h)$

$\Rightarrow$ LHL = -6

and RHL = $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (6x + 2)$

$\Rightarrow$ $RHL = \lim_{h \to 0} [6(3 + h) + 2]$

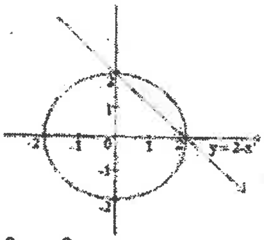
$\Rightarrow$ RHL = 20

$\because$ **LHL $\neq$ RHL**

$\therefore$ f is discontinuous at x = 3 Now, as f (x) is a polynomial function for x < -3 , -3< x < 3 and x > 3 so it is continuous in these intervals.

Hence, only x = 3 is the point of discontinuity of f(x).

Section D

32. 

$x^2 + y^2 = 4$ ...(1)

x + y = 2 ...(2)

From (2), y = 2 - x

Put this value of y in (1), we get,

$x^2 + (2 - x)^2 = 4$

$\Rightarrow 2x^2 - 4x = 0$

$\Rightarrow 2x(x - 2) = 0$

$\Rightarrow x = 0, 2$

When x = 0, y = 2 - 0 = 2

When x = 2, y = 2 - 2 = 0

$\therefore$ points of intersection are (0, 2) and (2, 0)

Required area = area of quadrant in first quadrant - area of triangle.

$= \int_0^2 \sqrt{4 - x^2}dx - \int_0^2 (2 - x)dx$

$= \int_0^2 \sqrt{(2)^2 - x^2dx} - \int_0^2 (2 - x)dx$

$= \left[\frac{x}{2}\sqrt{(2)^2 - x^2} + \frac{(2)^2}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[2x - \frac{x^2}{2}\right]_0^2$

$= \left[\frac{2}{2}\sqrt{4-4} + \frac{4}{2}\sin^{-1}1\right] - \left[0 + \frac{4}{2}\sin^{-1}0\right] - \left[\left(4 - \frac{4}{2}\right) - (0 - 0)\right]$

$= \left(0 + \frac{4}{2} \times \frac{\pi}{2}\right) - (0 + 2 \times 0) - 2 + 0 = \pi - 2$

33. A = R - {3}, B = R - {1}

f : A $\rightarrow$ B is defined as $f(x) = \left(\frac{x-2}{x-3}\right)$.

Let x, y $\in$ A such that f(x) = f(y).

$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$

$\Rightarrow$ (x - 2) (y - 3) = (y - 2) (x - 3)

$\Rightarrow$ xy - 3x - 2y + 6 = xy - 3y - 2x + 6

$\Rightarrow$ -3x - 2y = -3y - 2x

$\Rightarrow$ 3x - 2x = 3y - 2y

$\Rightarrow$ x = y

Therefore, f is one-one.

Let y $\in$ B = R - {1}

Then, y $\neq$ 1.

The function f is onto if there exists x $\in$ A such that f(x) = y.

Now, f (x) = y

$\Rightarrow \frac{x-2}{x-3} = y$

$\Rightarrow$ x - 2 = xy - 3y

$\Rightarrow$ x(1 - y) = -3y + 2

$\Rightarrow x = \frac{2-3y}{1-y} \in A \quad [y \neq 1]$

Thus, for any y $\in$ B, there exists $\frac{2-3y}{1-y} \in A$ such that

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$$f\left(\frac{2-3y}{1-y}\right)=\frac{\left(\frac{2-3y}{1-y}\right)-2}{\left(\frac{2-3y}{1-y}\right)-3}=\frac{2-3y-2+2y}{2-3y-3+3y}=\frac{-y}{-1}=y$$

∴ f is onto.

Hence, function f is one-one and onto.

OR

We observe the following properties of f.

Injectivity: Let x, y ∈ R_0 such that f(x) = f(y). Then,

$f(x)=f(y)\Rightarrow\frac{1}{x}=\frac{1}{y}\Rightarrow x=y$

So, f : $R_0 \to R_0$ is one-one.

Surjectivity: Let y be an arbitrary element of R_0 (co-domain) such that f(x) = y. Then,

$f(x)=y\Rightarrow\frac{1}{x}=y\Rightarrow x=\frac{1}{y}$

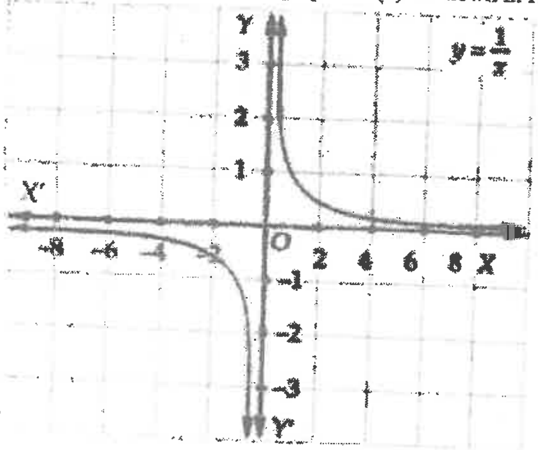
Clearly, $x=\frac{1}{y}\in R_0$ (domain) for all y ∈ R_0 (co-domain).

Thus, for each y ∈ R_0 (co-domain) there exits $x=\frac{1}{y}\in R_0$ (domain) such that $f(x)=\frac{1}{x}=y$

So, f : $R_0 \to R_0$ is onto.

Hence, f: $R_0 \to R_0$ is one-one onto.

This is also evident from the graph of f(x) as shown in fig.



Let us now consider f : N → R_0 given by $f(x)=\frac{1}{x}$

For any x, y ∈ N, we find that

f(x) = f(y) $\Rightarrow\frac{1}{x}=\frac{1}{y}\Rightarrow x=y$

So, f: N → R_0 is one-one.

We find that $\frac{2}{3},\frac{3}{5}$ etc. in co-domain R_0 do not have their pre-image in domain N. So, f: N →R_0 is not onto.

Thus, f: N →R_0 is one-one but not onto.

34.

|A| =-9

$$\text{adj}A=\begin{bmatrix}3 & 4 & -26\\ 3 & 1 & -11\\ -3 & -4 & 17\end{bmatrix}$$

$$A^{-1}=-1/9\begin{bmatrix}3 & 4 & -26\\ 3 & 1 & -11\\ -3 & -4 & 17\end{bmatrix}$$

$$X=A^{-1}B=-1/9\begin{bmatrix}-27\\ -18\\ 9\end{bmatrix}=\begin{bmatrix}3\\ 2\\ -1\end{bmatrix}$$

X=3, y=2, z= -1

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35. Let

$\vec{a} = l_1\hat{i} + m_1\hat{j} + n_1\hat{k}$

$\vec{b} = l_2\hat{i} + m_2\hat{j} + n_2\hat{k}$

$\vec{c} = l_3\hat{i} + m_3\hat{j} + n_3\hat{k}$

$\vec{d} = (l_1 + l_2 + l_3)\hat{i} + (m_1 + m_2 + m_3)\hat{j} + (n_1 + n_2 + n_3)\hat{k}$

Also, let α, β and γ are the angles between $\vec{a}$ and $\vec{d}$, $\vec{b}$ and $\vec{d}$, $\vec{c}$ and $\vec{d}$.

$\therefore \cos\alpha = l_1(l_1 + l_2 + l_3) + m_1(m_1 + m_2 + m_3) + n_1(n_1 + n_2 + n_3)$

$= l_1^2 + l_1l_2 + l_1l_3 + m_1^2 + m_1m_2 + m_1m_3 + n_1^3 + n_1n_2 + n_1n_3$

$= (l_1^2 + m_1^2 + n_1^2) + (l_1l_2 + l_1l_3 + m_1m_2 + m_1m_3 + n_1n_2 + n_1n_3)$

$= 1 + 0 = 1$

$[\because l_1^2 + m_1^2 + n_1^2 = 1$ and $l_1 \perp l_2, l_1 \perp l_3, m_1 \perp m_2, m_1 \perp m_3, n_1 \perp n_2, n_1 \perp n_3]$

Similarly, $\cos\beta = l_2(l_1 + l_2 + l_3) + m_2(m_1 + m_2 + m_3) + n_2(n_1 + n_2 + n_3)$

$= 1 + 0 = 1$ and $\cos\gamma = 1 + 0 = 0$

$\Rightarrow \cos\alpha = \cos\beta = \cos\gamma$

$\Rightarrow \alpha = \beta = \gamma$

So, the line whose direction cosines are proportional to $l_1 + l_2 + l_2$, $m_1 + m_2 + m_3$, $n_1 + n_2 + n_3$ make equal to angles with the three mutually perpendicular lines whose direction cosines are l_1, m_1, n_1, l_2, m_2, n_2 and l_3, m_3, n_3 respectively.

OR

Equation of line in vector form

Line I: $\vec{r} = (\hat{i} - \hat{j} + 0\hat{k}) + \lambda(2\hat{i} + 0\hat{j} + \hat{k})$

Line II: $\vec{r} = (2\hat{f} - \hat{j}) + \mu(\hat{i} + \hat{j} - \hat{k})$

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Here,

$\overrightarrow{a_1} = \hat{\imath} - \hat{\jmath} + 0\hat{k}$

$\overrightarrow{a_2} = 2\hat{\imath} - \hat{\jmath}$

$\overrightarrow{b_1} = 2\hat{i} + 0\hat{j} + \hat{k}$

$\overrightarrow{b_2} = \hat{i} + \hat{j} - \hat{k}$

We know that the shortest distance between lines is

$d = \frac{|(\overrightarrow{a_2}-\overrightarrow{a_1})(\overrightarrow{b_1}\times\overrightarrow{b_2})|}{|\overrightarrow{b_1}\times\overrightarrow{b_2}|}$

$(\overrightarrow{a_2} - \overrightarrow{a_1}) = (2\hat{\imath} - \hat{\jmath}) - (\hat{\imath} - \hat{\jmath} + 0\hat{k})$

$(\overrightarrow{a_2} - \overrightarrow{a_1}) = \hat{\imath} + 0\hat{\jmath} + 0\hat{k}$

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

$\vec{b}_1 \times \vec{b}_2 = (0 - 1)\hat{i} - (-2 - 1)\hat{j} + (2 - 0)\hat{k}$

$\Rightarrow \vec{b}_1 \times \vec{b}_2 = -\hat{i} + 3\hat{j} + 2\hat{k}$

$|\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{(-1)^2 + 3^2 + 2^2}$

$\Rightarrow |\vec{b}_1 \times \vec{b}_2| = \sqrt{14}$

$|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = |(\hat{\imath} + 0\hat{\jmath} + 0\hat{k})(-\hat{\imath} + 3\hat{\jmath} + 2\hat{k})|$

$\Rightarrow |(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = 1$

Substituting these values in the expression,

$d = \frac{|(\vec{a}_2-\vec{a}_1)(\vec{b}_1\times\vec{b}_2)|}{|\vec{b}_1\times\vec{b}_2|}$

$d = \frac{1}{\sqrt{14}}$

$d = \frac{1}{\sqrt{14}}$ units

Shortest distance d between the lines is not 0. Hence the given lines are not intersecting.

Section E

36. i. Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.

$\therefore P(E) = \frac{50}{100} = \frac{1}{2}$, $P(M) = \frac{35}{100} = \frac{7}{20}$ and $P(E \cap M) = \frac{25}{100} = \frac{1}{4}$

The probability that the selected student has failed in Economics if it is known that he has failed in Mathematics.

Required probability = $P(\frac{E}{M})$

$= \frac{P(E\cap M)}{P(M)} = \frac{\frac{1}{4}}{\frac{7}{20}} = \frac{1}{4} \times \frac{20}{7} = \frac{5}{7}$

ii. Let E denote the event that student has failed in Economics and M denote the event that student has failed in Mathematics.

$\therefore P(E) = \frac{50}{100} = \frac{1}{2}$, $P(M) = \frac{35}{100} = \frac{7}{20}$ and $P(E \cap M) = \frac{25}{100} = \frac{1}{4}$

The probability that the selected student has failed in Mathematics if it is known that he has failed in Economics.

Required probability = P(M/E)

$= \frac{P(M\cap E)}{P(E)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$

iii. Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.

$\therefore P(E) = \frac{50}{100} = \frac{1}{2}$, $P(M) = \frac{35}{100} = \frac{7}{20}$ and $P(E \cap M) = \frac{25}{100} = \frac{1}{4}$

The probability that the selected student has passed in Mathematics if it is known that he has failed in Economics

Required probability = P(M'/E)

$\Rightarrow P(M'/E) = \frac{P(M'\cap E)}{P(E)}$

$= \frac{P(E)-P(E\cap M)}{P(E)}$

$= \frac{\frac{1}{2}-\frac{1}{4}}{\frac{1}{2}}$

$\Rightarrow P(M'/E) = \frac{1}{2}$

OR

Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.

$\therefore P(E) = \frac{50}{100} = \frac{1}{2}$, $P(M) = \frac{35}{100} = \frac{7}{20}$ and $P(E \cap M) = \frac{25}{100} = \frac{1}{4}$

The probability that the selected student has passed in Economics if it is known that he has failed in Mathematics

Required probability = P(E'/M)

$\Rightarrow P(E'/M) = \frac{P(E' \cap M)}{P(M)}$

$= \frac{P(M) - P(E \cap M)}{P(M)}$

$= \frac{\frac{7}{20} - \frac{1}{4}}{\frac{7}{20}} \Rightarrow P(E'/M) = \frac{2}{7}$

37. i. Total displacement = $|\vec{d_1}| + |\vec{d_2}| + |\vec{d_3}|$

$|\vec{d_1}| = \sqrt{6^2 + 8^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

= 10 km

$|\vec{d_2}| = \sqrt{3^2 + 4^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

= 5 km

$|\vec{d_3}| = \sqrt{7^2 + 12^2}$

$= \sqrt{49 + 144}$

= 13.89

Total displacement = 10 + 5 + 13.89

= 28.89

≈ 29 km

ii. Speed = $= \frac{\text{Total distance}}{\text{Total time}}$

$= \frac{28.89}{1.5}$

= 19.26 km/hr

iii. Displacement form village to zoo = $d_1 + d_2$

= 10 + 5

= 15 km

OR

Displacement from temple to mall = $d_2 + d_3$

= 5 + 13.89

= 18.89

≈ 19 km

38. i. $f(x) = -0.1x^2 + mx + 98.6$, being a polynomial function, is differentiable everywhere, hence, differentiable in (0, 12).

ii. $f(x) = -0.2x + m$

At Critical point

$0 = -0.2 \times 6 + m$

m = 1.2

iii. $f(x) = -0.1x^2 + 1.2x + 98.6$

$f'(x) = -0.2x + 1.2 = -0.2(x - 6)$

In the Interval	f'(x)	Conclusion
(0, 6)	+Ve	f is strictly increasing in [0, 6]
(6, 12)	-Ve	f is strictly decreasing in [6, 12]

OR

$f(x) = -0.1x^2 + 1.2x + 98.6$,

$f'(x) = -0.2x + 1.2$, $f'(6) = 0$,

$f''(x) = -0.2$

$f''(6) = -0.2 < 0$

Hence, by second derivative test 6 is a point of local maximum. The local maximum value = $f(6) = -0.1 \times 6^2 + 1.2 \times 6 + 98.6 = 102.2$

We have $f(0) = 98.6$, $f(6) = 102.2$, $f(12) = 98.6$

6 is the point of absolute maximum and the absolute maximum value of the function = 102.2.

0 and 12 both are the points of absolute minimum and the absolute minimum value of the function = 98.6.

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